Brane-induced gravity in warped backgrounds and the absence of the radion

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Abstract

The Randall-Sundrum model with brane-localized curvature terms is considered. It is shown that this model possesses some interesting features, in particular, the radion field is absent in it. Although there is no modification of gravity at long distances, the model predicts deviations from Newton's law at short distances. This effect can be observed in the experiments for testing gravity at sub-millimeter scales.

Keywords: Kaluza-Klein theories, branes, induced gravity

1 Introduction

Models with brane-localized curvature terms have been widely discussed in the literature during the last few years. In paper [1] it was argued that matter on the brane can induce a brane-localized curvature term via the quantum corrections, which appears in the low-energy effective action. An interesting feature of this model is a modification of gravity at ultra-large scales, which can be very interesting from the cosmological point of view. But later it was shown [2, 3] that there exists a strong coupling effect in this model, which makes it unacceptable. There were attempts to merge the DGP-proposal and models with warped backgrounds to get a long-distance modification of gravity. But it turnes out that such models must be rejected for some reasons (see, for example, [2, 4, 5]). For example, the Randall-Sundrum model with brane-localized curvature terms admits a long-distance modification of gravity, but in this case either the radion, the graviton or both fields become ghosts.

It would be interesting to consider such models from another point of view. A modification of gravity at large distances is not the only interesting effect, which can arise in the models. For example, in papers [6, 7] the spectrum of Kaluza-Klein gravitons in the RS background with brane-localized terms for different values of parameters was studied and some experimental constraints were found (for example, for collider experiments), but the radion field was not taken into account. As it was noted above, this field plays an important role in the spectrum of gravitational fluctuations and its existence can change some parameters of the model considerably to make it acceptable from the phenomenological point of view. One can recall the original Randall-Sundrum model with two branes [8], in which it is necessary to stabilize the size of extra dimension and to make the radion field massive (see [9, 10]), for example, with the help of the Goldberger-Wise mechanism [11].

In the present paper we study a model with brane-localized curvature terms, which is based on the Randall-Sundrum solution for the background metric. We will show that, with appropriate parameters, the model reproduces 4-dimensional gravity on the brane, does not contradict the known experimental data and provides some interesting consequences.

2 The setup

Let us choose the action of the model in the following form

$$S = S_g + S_1 + S_2, \tag{1}$$

where S_g, S_1 and S_2 are given by

$$\begin{aligned} S_g &= \frac{1}{16\pi\hat{G}} \int_E (R - \Lambda) \sqrt{-g}\, d^4x dy, \\ S_1 &= \frac{\alpha_1}{16\pi\hat{G}} \int_E \sqrt{-\tilde{g}}(\tilde{R} - \Lambda_1)\delta(y) d^4x dy, \\ S_2 &= \frac{\alpha_2}{16\pi\hat{G}} \int_E \sqrt{-\tilde{g}}(\tilde{R} - \Lambda_2)\delta(y - R) d^4x dy. \end{aligned} \tag{2}$$

Here $\tilde{g}_{\mu\nu}$ is the induced metric on the branes and the subscripts 1 and 2 label the branes. The model possesses the usual Z_2 orbifold symmetry. We also note that the signature of the metric g_{MN} is chosen to be $(-,+,+,+,+)$. Obviously, the model admits the Randall-Sundrum solution for the metric, which has the form

$$ds^2 = \gamma_{MN} dx^M dx^N = \gamma_{\mu\nu} dx^\mu dx^\nu + dy^2, \tag{3}$$

where $\gamma_{\mu\nu} = e^{2\sigma(y)}\eta_{\mu\nu}$, $\eta_{\mu\nu}$ is the Minkowski metric and the function $\sigma(y) = -k|y|$ in the interval $-R \le y \le R$. The parameter k is positive and has the dimension of mass, the parameters Λ and $\Lambda_{1,2}$, $\alpha_{1,2}$ are related to it as follows:

$$\Lambda = -k\alpha_1\Lambda_1 = k\alpha_2\Lambda_2 = -12k^2. \tag{4}$$

The function σ has the properties

$$\partial_4\sigma = -k\, sign(y), \quad \partial_4^2\sigma = -2k(\delta(y) - \delta(y - R)) \equiv -2k\tilde{\delta}. \tag{5}$$

The parameters α_1 and α_2 are not specified by the solution, and their possible values will be duscussed below.

We denote $\hat{\kappa} = \sqrt{16\pi\hat{G}}$, where $\hat{G}$ is the five-dimensional gravitational constant, and parameterize the metric g_{MN} as

$$g_{MN} = \gamma_{MN} + \hat{\kappa} h_{MN}, \tag{6}$$

h_{MN} being the metric fluctuations. In papers [9, 10] the second variation Lagrangian for the fluctuations of metric in the Randall-Sundrum model was obtained. In the case under consideration the presence of the brane-localized curvature terms changes this Lagrangian, and the addition can be easily calculated. But even with this addition the corresponding action is invariant under the gauge transformations

$$h'_{MN}(x,y) = h_{MN}(x,y) - (\nabla_M\xi_N(x,y) + \nabla_N\xi_M(x,y)), \tag{7}$$

where ∇_M is the covariant derivative with respect to the background metric γ_{MN}, and the functions $\xi_N(x,y)$ satisfy the orbifold symmetry conditions

$$\begin{aligned} \xi^\mu(x,-y) &= \xi^\mu(x,y), \\ \xi^4(x,-y) &= -\xi^4(x,y). \end{aligned} \tag{8}$$

With the help of these gauge transformations we can impose the gauge

$$h_{\mu 4} = 0,\ h_{44} = h_{44}(x) \equiv \phi(x), \tag{9}$$

which will be called the *unitary gauge* (see [9]). We would like to emphasize once again that the branes remain straight in this gauge, i.e. we *do not* use the bent-brane formulation, which allegedly destroys the structure of the model (this problem was discussed in [12]).

First, let us consider the case, where there is no matter on the branes. In this case the equations of motion for different components of the metric fluctuations in the unitary gauge take the form:

1) $\mu\nu$-component

$$\begin{aligned} &\frac{1}{2}\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \partial_4^2 h_{\mu\nu}\right) - \\ -\ & 2k^2 h_{\mu\nu} + \frac{1}{2}\partial_\mu\partial_\nu\tilde{h} + \frac{1}{2}\partial_\mu\partial_\nu\phi + \\ +\ & \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h} - \partial_4^2\tilde{h} - 4\partial_4\sigma\partial_4\tilde{h} - \partial_\rho\partial^\rho\phi + 12k^2\phi\right) + \\ +\ & \left[2kh_{\mu\nu} - 3k\gamma_{\mu\nu}\phi\right]\tilde{\delta} + \\ +\ & \frac{\alpha_i}{2}\delta_i\left[\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \partial_\mu\partial_\nu\tilde{h}\right) + \right. \\ +\ & \left.\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h}\right)\right] = 0, \end{aligned} \tag{10}$$

where $i = 1,2$, $\delta_1 = \delta(y)$, $\delta_2 = \delta(y-R)$

2) $\mu 4$-component,

$$\partial_4(\partial_\mu\tilde{h} - \partial^\nu h_{\mu\nu}) - 3\partial_4\sigma\partial_\mu\phi = 0, \tag{11}$$

which plays the role of a constraint,

3) 44-component

$$\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} + 6k^2\phi = 0, \tag{12}$$

where $\tilde{h} = \gamma^{\mu\nu}h_{\mu\nu}$. In what follows, we will also use an auxiliary equation, which is obtained by multiplying the equation for 44-component by 2 and subtracting it from the contracted equation for $\mu\nu$-component. This equation contains $\tilde{h}$ and ϕ only and has the form:

$$\begin{aligned} &\partial_4^2\tilde{h} + 2\partial_4\sigma\partial_4\tilde{h} - 8k^2\phi + 8k\phi\tilde{\delta} + \partial_\mu\partial^\mu\phi - \\ -\ & \frac{2}{3}\alpha_i\delta_i\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h}\right) = 0. \end{aligned} \tag{13}$$

Equation (4) suggests that there exist preferred values of the parameters α_i. Namely, if we choose these parameters to be

$$\alpha_1 = -\frac{1}{k}, \quad \alpha_2 = \frac{1}{k}, \tag{14}$$

the values of the cosmological constants on the branes Λ_i coincide with the cosmological constant in the bulk Λ. For our choice of the parameters brane 1 has a positive energy density, whereas brane 2 has a negative one. We note that one does not need to worry about the negative sign of the parameter α_1: we will see that the model is stable and does not contain tachyons or ghosts. In fact, our choice of α_i does not introduce any new dimensional parameter and does not contradict the naturalness condition.

An interesting observation is that, with conditions (14), equations (10), (11), (12) and (13) possess an additional symmetry under the transformations

$$h_{\mu\nu}(x,y) \to h_{\mu\nu}(x,y) + \sigma\gamma_{\mu\nu}\varphi(x) + \frac{1}{2k^2}\left(\sigma + \frac{1}{2}\right)\partial_\mu\partial_\nu\varphi(x), \tag{15}$$

$$\phi(x) \to \phi(x) + \varphi(x), \tag{16}$$

which *do not* belong to the gauge transformations (7). It is evident that with the help of these transformations we can impose the condition $\phi(x) \equiv 0$. At the first glance this symmetry seems to be rather strange. But let us have a look at equation (12). It implies that the second variation Lagrangian of the theory does not contain the kinetic term for the field $h_{44} = \phi(x)$ (even in the original Randall-Sundrum model). It means that the radion field can be regarded as an auxiliary field (recall the Supersymmetry). Therefore, there is no contradiction that for some values of parameters of the model this field can be totally eliminated from the theory. It should be noted that the symmetry (15) and (16) of the linearized equations of motion can correspond to some general symmetry of the action (1), which is not evident at first sight (but this is not necessarily so). As we will see later, if there is matter on the brane, there appears a scalar field due to the existence of the extra dimension, which *cannot* be identified with the 44-component of the metric fluctuations.

It should be noted, that the elimination of the radion in the Randall-Sundrum model with brane-localized curvature terms was discussed earlier, for example, in [2] and [5] (it is evident that since there exists some range of parameters for which the coefficient in front of the kinetic term for the radion could be either positive or negative, there exist some values of the parameters for which the radion is absent at all). But in this paper the equations of motion for the model with warped background are treated much more thoroughly, a convenient gauge is used and these equation are solved *exactly*.

Thus, we can consider the equations of motion without the radion. With the help of the regularization

$$\partial_4\sigma(\partial_4^2\sigma) = \frac{1}{2}\partial_4\left((\partial_4\sigma)^2\right) = \frac{1}{2}\partial_4 k^2 = 0, \tag{17}$$

we get from equations (12) and (13)

$$\partial_4\tilde{h} = const \cdot e^{-2\sigma}. \tag{18}$$

Let us consider Fourier expansion of all terms of equation (18) with respect to coordinate y. Since the term with the derivative ∂_4 has no zero mode, this equation implies that

$$\partial_4(e^{-2\sigma}h) = 0, \tag{19}$$

where $h = h_{\mu\nu}\eta^{\mu\nu}$. The residual gauge transformations are sufficient to impose the transverse-traceless gauge on the field $h_{\mu\nu}$ (see [9])

$$\begin{aligned} \partial^\nu h_{\mu\nu} &= 0, \\ h &= 0. \end{aligned} \tag{20}$$

Thus, the $\mu\nu$-equation takes the form ($\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$)

$$\frac{1}{2}\left(e^{-2\sigma}\Box h_{\mu\nu} + \partial_4^2 h_{\mu\nu}\right) - 2k^2 h_{\mu\nu} + 2k\tilde{\delta}h_{\mu\nu} - \frac{1}{2k}\tilde{\delta}e^{-2\sigma}\Box h_{\mu\nu} = 0. \tag{21}$$

It is not difficult to solve this equation. The zero mode has the form

$$h^0_{\mu\nu} = \alpha_{\mu\nu}(x)e^{2\sigma}, \tag{22}$$

whereas the massive modes have the form

$$\begin{aligned} h^m_{\mu\nu} = b^m_{\mu\nu}(x)\Psi^m(y), \quad \Box b^m_{\mu\nu}(x) = m^2 b^m_{\mu\nu}(x), \\ \Psi^m(y) = AJ_2\left(\frac{m}{k}e^{-\sigma}\right) + BN_2\left(\frac{m}{k}e^{-\sigma}\right), \end{aligned} \tag{23}$$

$J_2(t)$ and $N_2(t)$ being the Bessel and Neumann functions.

The term with δ-functions can be taken into account by imposing the boundary condition

$$AJ_0\left(\frac{m}{k}e^{-\sigma}\right) + BN_0\left(\frac{m}{k}e^{-\sigma}\right) = 0$$

at $y = 0$ and $y = R$. The first boundary condition can be satisfied by an appropriate choice of the coefficients A and B:

$$\Psi^m(y) = N_m\left(N_0\left(\frac{m}{k}\right)J_2\left(\frac{m}{k}e^{-\sigma}\right) - J_0\left(\frac{m}{k}\right)N_2\left(\frac{m}{k}e^{-\sigma}\right)\right), \tag{24}$$

where N_m is the norm of the eigenfunction. The second boundary condition, at $y = R$, defines the mass spectrum of the theory and can be rewritten as

$$N_0\left(\frac{m}{k}\right)J_0\left(\frac{m}{k}e^{kR}\right) - J_0\left(\frac{m}{k}\right)N_0\left(\frac{m}{k}e^{kR}\right) = 0. \tag{25}$$

One can see, that it is analogous to the one obtained in [9]. There exists a theorem about such combinations of products of Bessel and Neumann functions, which asserts that for $e^{kR} > 1$ this combination is an even function of m/k and its zeros are real and simple [13]. Thus, one does not need to worry about the stability of the system: there are no tachyons.

The normalized functions $\Psi^m(y)$ satisfy the equation (see, for example, [7])

$$\int dy e^{-2\sigma}\left[1 - \frac{1}{k}\delta(y) + \frac{1}{k}\delta(y-R)\right]\Psi^m\Psi^n = \delta_{mn}. \tag{26}$$

Let us calculate the norm of the zero mode eigenfunction $\Psi^0(y) = N_0 e^{2\sigma}$. Substituting it into (26) one can find that $\Psi^0(y)$ can not be normalized, because the left part of the equation (26) is equal to zero for arbitrary N_0. In other words, the second variation Lagrangian of the theory does not contain the kinetic term for the massless graviton, i.e. it is absent in this model. One can ask, why do we consider the model, which does not contain long-range gravity? But as we will see in the next section, the situation is rather different, if we place matter on the brane.

3 Matter on the brane

Let us suppose that there is matter on one of the branes (we will specify, which brane to choose later). Following the DGP-proposal [1], this matter induces a brane-localized term

$$S_{ind} = \frac{\Omega_{ind}^2}{k16\pi\hat{G}} \int d^4x\sqrt{\tilde{g}}\tilde{R}, \tag{27}$$

where Ω_{ind} is a dimensionless parameter.

Now let us discuss, on which brane we can put the matter (it is evident that the term (27) on a brane leads to the redefinition of the corresponding parameter α_i, for example, for brane 1 one gets $\alpha_1 = -\frac{1}{k} \to \alpha_1 = \frac{1}{k}(\Omega_{ind}^2 - 1)$). The problem is that the additional term (27) changes equation (21) and equation (25) for the eigenvalues, and there may appear tachyonic modes. This situation was discussed in detail in paper [14] (see also [15]). It was shown in these papers that the gravitational tachyons can be avoided, *at least* if (in our notations used in (1))

$$\alpha_1 \geq 0, \alpha_2 \geq 0 \tag{28}$$

(it was shown above that tachyons are absent in the case $\alpha_1 = -\frac{1}{k}$ and $\alpha_2 = \frac{1}{k}$ too). So if $\Omega_{ind} >> 1$ and the term (27) arises on the brane 1 (at y=0), the conditions (28) are satisfied and we do not need to worry about the stability of the model. We would also like to note that coordinates x are Galilean on brane 1, so that all the results obtained in coordinates x for brane 1 are correct from the physical point of view.

One can easily check that the term (27) does not violate the symmetry (15), (16) (because of the fact that $\sigma|_{y=0}$=0), but only if it arises on brane 1. Thus, the radion is absent in this model, and we can forget about the radion as a ghost. On the contrary, the term (27) on brane 2 (at $y = R$) violates the symmetry (15), (16). Moreover, the existence of the induced terms (27) on both branes makes the radion to be a ghost (see [5, 14]). One can also consider the case $\sigma = kR - k|y|$ and the existence of matter on brane 2 only (in this case coordinates x are Galilean on brane 2). The radion can be eliminated in this case too (since $\sigma|_{y=R} = 0$), but since $\alpha_1 = -\frac{1}{k} < 0$ and $\alpha_2 > \frac{1}{k}$ there may appear gravitational tachyons (see [14, 15]). Thus, the only physically relevant case, in which tachyons and ghost are absent (and the symmetry (15), (16) is preserved) is when the matter (and the induced term (27)) exists *on brane 1 only*. Brane 2 can be interpreted as a "naked" brane, i.e. a brane without matter on it.

Taking into consideration (27), we get new equations of motion (in the case $\phi(x) \equiv 0$)

1) $\mu\nu$-component

$$\begin{aligned} & \frac{1}{2}\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \partial_4^2 h_{\mu\nu}\right) - 2k^2 h_{\mu\nu} + \frac{1}{2}\partial_\mu\partial_\nu\tilde{h} + \\ + & \; 2k h_{\mu\nu}\tilde{\delta} + \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h} - \partial_4^2\tilde{h} - 4\partial_4\sigma\partial_4\tilde{h}\right) - \\ - & \; \frac{1}{2k}\tilde{\delta}\left[\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \partial_\mu\partial_\nu\tilde{h}\right) + \right. \\ + & \left. \; \gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h}\right)\right] + \\ + & \; \frac{\Omega_{ind}^2}{2k}\delta(y)\left[\left(\partial_\rho\partial^\rho h_{\mu\nu} - \partial_\mu\partial^\rho h_{\rho\nu} - \partial_\nu\partial^\rho h_{\rho\mu} + \partial_\mu\partial_\nu\tilde{h}\right) + \right. \\ + & \left. \; \gamma_{\mu\nu}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h}\right)\right] = -\frac{\hat{\kappa}}{2}\delta(y)t_{\mu\nu}(x), \end{aligned} \tag{29}$$

2) $\mu 4$-component,

$$\partial_4(\partial_\mu\tilde{h} - \partial^\nu h_{\mu\nu}) = 0, \tag{30}$$

3) 44-component

$$\frac{1}{2}(\partial^\mu\partial^\nu h_{\mu\nu} - \partial_\mu\partial^\mu\tilde{h}) - \frac{3}{2}\partial_4\sigma\partial_4\tilde{h} = 0, \tag{31}$$

and

$$\begin{aligned} & \partial_4^2\tilde{h} + 2\partial_4\sigma\partial_4\tilde{h} + \frac{2}{3k}\tilde{\delta}\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h}\right) - \\ - & \; \frac{2}{3k}\Omega_{ind}^2\delta(y)\left(\partial^\rho\partial^\sigma h_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{h}\right) = \frac{\hat{\kappa}}{3}\delta(y)t_\mu^\mu(x), \end{aligned} \tag{32}$$

where $t_{\mu\nu}$ is the energy-momentum tensor of matter on the brane, for example, of a static point-like mass. Since $\sigma|_{y=0} = 0$, the existence of matter on brane 1 does not violate the symmetry (15), (16).

To solve these equations, it is convenient to make the following substitution

$$h_{\mu\nu}(x,y) = b_{\mu\nu}(x,y) + \frac{1}{2k^2}\partial_\mu\partial_\nu f(x). \tag{33}$$

We note that the second term in substitution (33) is a pure gauge from the four-dimensional point of view from the brane.

Let us take equation (32). Multiplying it by $e^{2\sigma}$, using equation (31), taking into account regularization (17) and the fact that the terms with derivative have no zero mode, we get

$$\partial^\rho\partial^\sigma b_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{b} = -\frac{\hat{\kappa}k}{2\Omega_{ind}^2}\,t_\mu^\mu, \tag{34}$$

$$\partial^\rho\partial^\sigma b_{\rho\sigma} - \partial_\rho\partial^\rho\tilde{b} + 3\partial_\rho\partial^\rho f = 0, \tag{35}$$

and

$$\partial_4\tilde{b} = 0, \tag{36}$$

as it was made in Section 2. With the help of the $\mu 4$-component of the equations one can impose the de Donder gauge on the field $b_{\mu\nu}$ (see also [10])

$$\partial^{\nu}\left(b_{\mu\nu}-\frac{1}{2}\gamma_{\mu\nu}\tilde{b}\right)=0. \tag{37}$$

It follows from equations (34), (35) and (37) that

$$\Box f=\frac{\hat{\kappa}k}{6\Omega_{ind}^{2}}\,t. \tag{38}$$

and

$$\Box\tilde{b}=\frac{\hat{\kappa}k}{\Omega_{ind}^{2}}\,t, \tag{39}$$

where $t=\eta^{\mu\nu}t_{\mu\nu}$. We would also like to note that it is not correct to identify the field f with the radion field ϕ, which is the 44-component of the metric fluctuations. But it is apparent that this scalar part of $\mu\nu$-component of the metric fluctuations exists due to the existence of the extra dimension and appears only if there is some matter on the brane. It is similar to what happens in the RS2 model, where the radion field appears, if we place matter on the brane (see [16]).

Substituting (33) into (29), we get

$$\begin{aligned}
&\frac{1}{2}\left(e^{-2\sigma}\Box\left[b_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}b\right]+\partial_{4}^{2}b_{\mu\nu}\right)-2k^{2}b_{\mu\nu}+2kb_{\mu\nu}\tilde{\delta}- \\
-&\ \frac{1}{2k}\tilde{\delta}\left(e^{-2\sigma}\Box\left[b_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}b\right]\right)+\frac{\Omega_{ind}^{2}}{2k}\delta(y)\left(e^{-2\sigma}\Box\left[b_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}b\right]\right)= \\
=&\ -\frac{\hat{\kappa}}{2}\delta(y)t_{\mu\nu}+\left(1-\frac{\tilde{\delta}}{k}\right)\left[\partial_{\mu}\partial_{\nu}f-\eta_{\mu\nu}\Box f\right],
\end{aligned} \tag{40}$$

where f is defined by (38).

It would be interesting to get the equation for the zero mode of $h_{\mu\nu}$. As it was noted above, the zero mode has the form $h_{\mu\nu}^{0}=\alpha_{\mu\nu}e^{2\sigma}$. Let us multiply equation (40) by $e^{2\sigma}$ and integrate it over coordinate y. Using the orthonormality conditions, which is modified by the term (27) to be

$$\int dye^{-2\sigma}\left[1-\frac{1}{k}\delta(y)+\frac{1}{k}\delta(y-R)+\frac{\Omega_{ind}^{2}}{k}\delta(y)\right]\Psi^{m}\Psi^{n}=\delta_{mn}, \tag{41}$$

we get

$$\Box\left(\alpha_{\mu\nu}-\frac{1}{2}\eta_{\mu\nu}\alpha\right)=-\frac{\hat{\kappa}k}{\Omega_{ind}^{2}}\,t_{\mu\nu}. \tag{42}$$

An analogous procedure was made in the case of RS1 model in paper [10].

A very interesting thing happens: the massless graviton reappears in the model. It looks as if the matter "produces" the massless gravity via the induced term (which appears if there is matter on the brane) for itself. Thus, the gravity on the brane in the zero mode approximation is defined by $h_{\mu\nu}^{0}|_{y=0}=\alpha_{\mu\nu}$. The four-dimensional gravitational constant is defined by the parameter Ω_{ind} (i.e. by the induced term) instead of the factor e^{2kR} in the original RS1 model.

4 Massive modes

Thus, we have found a solution for the gravity in the zero mode approximation. Now let us estimate the effects, which can be produced by the massive modes. We will not solve equation (40), as it was made in [10] for the RS1 model. Let us estimate the masses of the lowest modes and their wave functions.

An analogue of equation (21) in the presence of the term (27) has the form

$$\begin{aligned} & \frac{1}{2}\left(e^{-2\sigma}\Box h_{\mu\nu}+\partial_4^2 h_{\mu\nu}\right)-2k^2 h_{\mu\nu}+2k\tilde{\delta}h_{\mu\nu}-\frac{1}{2k}\tilde{\delta}e^{-2\sigma}\Box h_{\mu\nu}+ \\ + \quad & \frac{\Omega_{ind}^2}{2k}\delta(y)e^{-2\sigma}\Box h_{\mu\nu}=0. \end{aligned} \tag{43}$$

Following in the footsteps of Section 2, we arrive at the following relations:

$$\Psi^m(y)=N_m\left(N_0\left(\frac{m}{k}e^{kR}\right)J_2\left(\frac{m}{k}e^{-\sigma}\right)-J_0\left(\frac{m}{k}e^{kR}\right)N_2\left(\frac{m}{k}e^{-\sigma}\right)\right), \tag{44}$$

where N_m is a normalization constant, and

$$\begin{aligned} & N_0\left(\frac{m}{k}e^{kR}\right)J_0\left(\frac{m}{k}\right)-J_0\left(\frac{m}{k}e^{kR}\right)N_0\left(\frac{m}{k}\right)+ \\ + \quad & \Omega_{ind}^2\left[N_0\left(\frac{m}{k}e^{kR}\right)J_2\left(\frac{m}{k}\right)-J_0\left(\frac{m}{k}e^{kR}\right)N_2\left(\frac{m}{k}\right)\right]=0. \end{aligned} \tag{45}$$

Let us choose kR such that e^{kR} is of the order $10^2 \div 10^3$. We can make this assumption, since it is not necessary to solve the hierarchy problem with the help of the factor e^{kR} - the four-dimensional Planck mass is defined by Ω_{ind}. Since Ω_{ind} is assumed to be much larger than 1 (we want to have a small five-dimensional Planck mass), the masses of the lowest modes are defined by

$$J_0\left(\frac{m}{k}e^{kR}\right)=0. \tag{46}$$

Thus, $m_{low}\sim ke^{-kR}$.

Now let us estimate the normalization constants of the lowest modes. Using the fact that the Bessel and Neumann functions are of the order ~ 1 for the masses, which are the solutions of (46), from (41) we get

$$\frac{1}{N_m^2}=\left(\sim Re^{2kR}\right)-\frac{1}{k}Z_m^2(0)+\frac{1}{k}e^{2kR}Z_m^2(R)+\frac{\Omega_{ind}^2}{k}Z_m^2(0), \tag{47}$$

where $\Psi^m(y)=N_m Z_m(y)$. Since $\Omega_{ind}>>e^{2kR}$, we get

$$N_m\approx\frac{\sqrt{k}}{\Omega_{ind}Z_m(0)}. \tag{48}$$

Now it is not difficult to calculate the coupling constants of the massless and massive modes to matter on the brane ($N_0=\frac{\sqrt{k}}{\Omega_{ind}}$):

$$\frac{1}{2}\int_{brane}d^4x\left(\hat{\kappa}\frac{\sqrt{k}}{\Omega_{ind}}\alpha_{\mu\nu}(x)t^{\mu\nu}+\hat{\kappa}\sum_m\frac{\sqrt{k}}{\Omega_{ind}}b_{\mu\nu}^m(x)t^{\mu\nu}\right). \tag{49}$$

Identifying the combination $\frac{\Omega_{ind}}{\hat{\kappa}\sqrt{k}}$ with the four-dimensional Planck mass M_{Pl}, we get

$$\frac{1}{2}\int_{brane} d^4x \left(\frac{1}{M_{Pl}} \alpha_{\mu\nu}(x) t^{\mu\nu} + \frac{1}{M_{Pl}} \sum_m b^m_{\mu\nu}(x) t^{\mu\nu} \right). \tag{50}$$

We see that the coupling constant of the lowest massive modes is the same, as of the massless graviton.

Now let us discuss the parameters of the model, which can give some interesting experimental consequences. Taking $M_* \sim k$ in the eV range (M_* is the five-dimensional Planck mass) and $kR \sim 5$ we get the size of the extra dimension of the order of $10^{-5}cm$. The lowest modes have the masses of the order of $10^{-2}eV$, which correspond to the corrections to Newton's law with the strength of the massless graviton at the distances of the order of $10^{-3}cm$, which is the micrometer scale. It can be interesting for experiments on testing gravity at sub-millimeter scales.

5 Conclusion

In this paper a model with brane-localized terms, based on the Randall-Sundrum background solution for the metric, was considered. The original Randall-Sundrum scenario was used not for the solution of hierarchy problem, but as a "constructor" of the branes with tension. Thus, the proper gravitational field of the branes was taken into account. We showed that this model is free from tachyons and ghosts and provides some interesting effects, such as the absence of the radion and "inducing" the long-range gravity by matter on the brane. With an appropriate choice of the parameters, this model can lead to interesting experimental consequences.

One may ask: why not to choose the original Randall-Sundrum model with the DGP term ($\sim \Omega^2_{ind}$), for example, on brane 1 only? According to [5, 14] there are no gravitational tachyons and ghosts in this case (but the radion field exists because of the absence of the symmetry (15), (16)). But the answer to the question about the radion as a ghost can be obtained only after *the exact* solving the equations of motion for the linearized gravity (as it was made, for example, in [9] for the RS1 model). Nevertheless, if we retain strong gravity in the bulk (small M_*, otherwise all the constructions make no sense) and $\Omega_{ind} >> e^{2kR}$, the term (27) on brane 1 makes the radion to be a ghost (all the reasonings are the same as in paper [2] for the Tev brane in the Randall-Sundrum model)! But even if for some range of parameters the radion is not a ghost (we remind that this issue can become clear after solving the corresponding equations), the possible bounds on the coupling constant of the radion may create an additional restrictions on the choice of the parameters k and R. At the same time the case of $\alpha_2 = \frac{1}{k}$ provides the absence of the radion and more freedom in our choice of parameters.

We would also like to add that the accuracy in treating the equations of motion can provide some interesting results, which are not evident at the first sight.

It would be interesting to calculate corrections to Newton's potential by solving directly equation (40), as it was made in [10] for the RS1 model, but this problem deserves a further investigation.

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